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Long-distance effects and final state interactions in $B \to \pi\pi K$ and $B \to K\bar{K}K$ decays

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Abstract

B decays into $\pi\pi K$ and $K\bar{K}K$, where the $\pi\pi$ and $\bar{K}K$ pairs interact in isospin zero S-wave, are studied in the $\pi\pi$ effective mass range from threshold to 1.2 GeV. The interplay of strong and weak decay amplitudes is analyzed using an unitary $\pi\pi$ and $K\bar{K}$ coupled channel model. Final state interactions are described in terms of four scalar form factors constrained by unitarity and chiral perturbation theory. Branching ratios for the $B \to f_0(980)K$ decay, calculated in the factorization approximation with some QCD corrections, are too low as compared to recent data. In order to improve agreement with experiment, we introduce long-distance contributions called charming penguins. Effective mass distributions, branching ratios and asymmetries are compared with the existing data from BaBar and Belle Collaborations. A particularly large negative asymmetry in charged B decays is predicted for one set of the charming penguin amplitudes.

1. Introduction

Recent experimental results from B factories indicate that charmless hadronic three-body decays are more frequent than two-body ones [1]. Moreover one observes on Dalitz plots a definitive surplus of events at relatively small effective masses. This is a signal of especially strong interactions between hadrons at not too high relative energies. Many resonances are explicitly visible but in general the interference pattern is quite complicated. Knowledge of these final state interactions is important to obtain a precise determination of the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements. Weak decay observables give information on hadron–hadron interactions and internal quark or hadronic structure of the produced particles.

Prominent maxima in the $\pi^+\pi^-$ spectra are observed in the $B \to \pi^+\pi^-K$ decays in the $f_0(980)$ region [2–9]. The $B^+ \to f_0(980)K^+$ and $B^0 \to f_0(980)K^0$ branching ratios are relatively large, and

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of the order of 10^{-5}. Direct and time-dependent CP-violating asymmetries are also measured. The first Belle result on the $B^+ \to f_0(980)K^+$ branching ratio [2] has motivated the study of Chen [10]. In the perturbative QCD approach Chen finds that the non-strange content of the $f_0(980)$ can be important. According to Cheng and Yang [11], subleading corrections due to intrinsic gluon effects inside B meson may enhance the decay rate of $B \to f_0(980)K$. B decays into scalar–pseudoscalar or scalar–vector particles have been studied by Minkowski and Ochs with a special emphasis on the presence of the lightest glueball [12]. The $\pi^+\pi^-$ mass spectrum in $B \to K\pi\pi$ decays, reported by Belle in 2003, is reproduced by a model amplitude of the coherent sum of $f_0(980)$, $f_0(1500)$ and a very broad glueball as a background.

In the present Letter we study the B decays into $\pi\pi K$ and $K\bar{K}K$. We restrict ourselves to the case where the produced $\pi\pi$ or $K\bar{K}$ pairs interact in isospin zero S-wave from the $\pi\pi$ threshold to about 1.2 GeV. One expects the $\pi\pi$ isospin two S-wave contribution to be small since the upper limit of the branching fraction for the $B^+ \to \pi^+\pi^+K^-$ decay is less than 1.8×10^{-6} [13]. Using the $K\bar{K}/\pi\eta$ branching ratio of $a_0(980)$ [1] and the upper limit of 2.5×10^{-6} [14] for the branching ratio of $B^+ \to a_0^0(980)K^+$, $a_0^0(980) \to \pi^0\eta$, one can estimate the branching fraction $\mathcal{B}(B^+ \to a_0^0(980)K^+, a_0^0(980) \to K^+K^-)$ to be smaller than 1×10^{-6}. This indicates that the $K\bar{K}$ isospin one S-wave amplitude is suppressed in the $B^\pm \to K^+K^-K^\pm$ decays.

Two-pion S-wave rescattering effects have been recently considered by Gardner and Meißner [15]. They study the effect of the $f_0(600)$ (or σ) resonance on the B^0 decay into $\pi^+\pi^-\pi^0$ in the range where the $\rho(770)\pi$ channel dominates. The $\sigma\pi$ channel can play a role in the determination of the CKM angle α from the $B^0 \to \rho\pi$ decays. Gardner and Meißner describe the broad $f_0(600)$ introducing a scalar form factor constrained by the chiral dynamics of low-energy meson–meson interactions [16]. This scalar form factor is used instead of the commonly applied Breit–Wigner form to improve the description of the broad σ and the understanding of the $B \to \rho\pi$ decays.

We extend the approach of Ref. [15] to the $f_0(980)$ resonance. The four strange and non-strange $\pi\pi$ and $K\bar{K}$ scalar form factors are constrained by chiral perturbation theory as developed by Meißner and Oller [16]. Our final state interaction is, however, different from that of [15] and [16]. Here we consider the unitary $\pi\pi$ and $K\bar{K}$ coupled channel model of [17].

First the $B \to (\pi\pi)_{S\text{-wave}}K$, $B \to (K\bar{K})_{S\text{-wave}}K$ decay amplitudes are calculated within the naive factorization approximation [18,19]. Penguin amplitudes interfere destructively which leads to much too small $B \to f_0(980)K$ branching ratios. Then we consider some QCD factorization corrections [20] calculated by de Groot, Cottingham and Whittingham [21]. These corrections are not sufficient to obtain agreement with experiment. Further contributions are needed. Here we include the long-distance contributions which have been considered in [21] to improve their fit to hadronic charmless strange and non-strange two-body B-decay data. These amplitudes, called charming penguin terms, originate from enhanced charm quark loops [22]. They could, for instance, correspond to weak decays of B to intermediate $D_s^{(*)}D^{(*)}$ states followed by transitions to $f_0(980)K$ final states via $c\bar{c}$ annihilations. Their addition allows us to obtain a good agreement with the measured $B \to f_0(980)K$ branching fractions.

In Section 2 we describe our weak decay amplitudes supplemented by the scalar form factors. Our model for the final state interactions is given in Section 3. Results of calculations and comparison with available data are presented in Section 4. In Section 5 we give some conclusions and final remarks.

2. Amplitudes for the $B \to \pi\pi K$ and $B \to K\bar{K}K$ decays

We shall write the model amplitudes for the following decays: $B^\pm \to (\pi\pi)_S K^\pm$, $B^\pm \to (K\bar{K})_S K^\pm$, $B^0 \to (\pi\pi)_S K^0$, $B^0 \to (K\bar{K})_S K^0$, $\bar{B}^0 \to (\pi\pi)_S \bar{K}^0$ and $\bar{B}^0 \to (K\bar{K})_S \bar{K}^0$. Here by $(\pi\pi)_S$ and $(K\bar{K})_S$ we mean $\pi^+\pi^-$ or $\pi^0\pi^0$ and K^+K^- or $K^0\bar{K}^0$ pairs in isospin zero S-wave.

The possible quark line diagrams for the B^- decay, together with the final state mesons, are shown in Fig. 1. For the B^0 decay there are only two types of penguin diagrams similar to those shown in Fig. 1(b) and (c). The tree diagram of Fig. 1(a) is absent. The $u\bar{u}$ or $s\bar{s}$ transitions into $\pi\pi$ or $K\bar{K}$ states, shown in Fig. 1, are described by four scalar form factors.

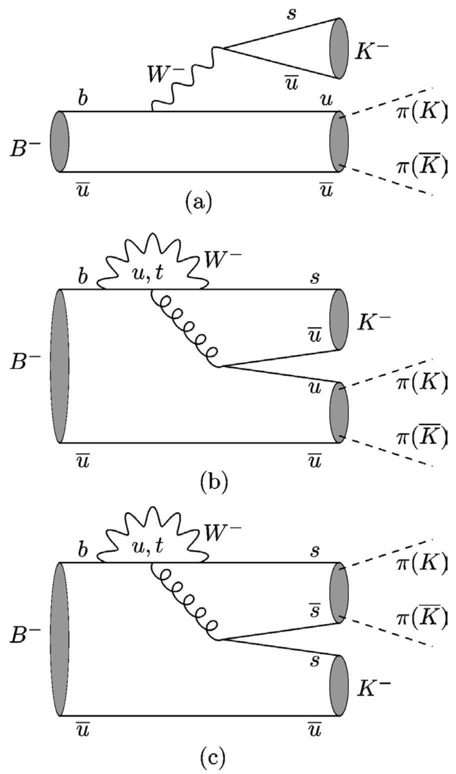

Fig. 1. Quark line diagrams for the B^- decay: (a) tree diagram, (b) and (c) penguin diagrams. The spring-like lines represent gluon exchange and the dashed ones the $\pi\pi$ or $K\bar{K}$ isospin zero S-wave pairs.

In the approximation used in our approach the effective weak Hamiltonian H is replaced by the sum of products of factorized currents [19]. We introduce some of the QCD factorization corrections and the charming penguin amplitudes considered in [21]. Then the $B^- \to (\pi^+\pi^-)_S K^-$ decay amplitude is

$$\langle (\pi^+\pi^-)_S K^- | H | B^- \rangle = \frac{G_F}{\sqrt{2}} \sqrt{\frac{2}{3}} \{ \chi [P(m_{\pi\pi})U + C(m_{\pi\pi})] \Gamma_1^{n*}(m_{\pi\pi}) + [Q(m_{\pi\pi})V + \chi C(m_K)] \Gamma_1^{s*}(m_{\pi\pi}) \}, \tag{1}$$

where G_F is the Fermi coupling constant and χ is a constant which will be estimated from the properties of the $f_0(980)$ decay. The functions $\Gamma_1^n(m_{\pi\pi})$ and $\Gamma_1^s(m_{\pi\pi})$ are the non-strange and strange pion scalar form factors depending on the effective pion-pion mass $m_{\pi\pi}$. Furthermore the functions $P(m_{\pi\pi})$ and $Q(m_{\pi\pi})$ defined as

$$P(m_{\pi\pi}) = f_K (M_B^2 - m_{\pi\pi}^2) F_0^{B\to(\pi\pi)_S}(M_K^2), \tag{2}$$

$$Q(m_{\pi\pi}) = \frac{2\sqrt{2}B_0}{m_b - m_s}(M_B^2 - M_K^2) F_0^{B\to K}(m_{\pi\pi}^2), \tag{3}$$

are proportional to the $B \to (\pi\pi)_S$ and $B \to K$ transition form factors, $F_0^{B\to(\pi\pi)_S}(M_K^2)$ and $F_0^{B\to K}(m_{\pi\pi}^2)$, respectively. The masses of B meson, kaon, pion, b-quark, strange-, down- and up-quarks are denoted by M_B, M_K, m_π, m_b, m_s, m_d and m_u, respectively. In Eq. (2) f_K is the kaon decay constant. In Eq. (3) B_0 is related to the vacuum quark condensate: $B_0 = -\langle 0|\bar{q}q|0\rangle / f_\pi^2$, f_π being the pion decay constant equal to 92.4 MeV. We use the formula $B_0 = m_\pi^2/(2\hat{m})$, where $\hat{m}$ is the average mass of the light quarks u and d. We put $\hat{m} = 5$ MeV and following Ref. [19] we take $m_s = 0.122$ GeV and $m_b = 4.88$ GeV. The functions U and V in (1) depend on the combinations of the coefficients a_i [18–21] and on the products of the CKM matrix elements $\lambda_u = V_{ub}V_{us}^*$ and $\lambda_t = V_{tb}V_{ts}^*$:

$$U = \lambda_u [a_1 + a_4^u - a_4^c + (a_6^c - a_6^u) r] + \lambda_t (a_6^c r - a_4^c), \tag{4}$$

$$V = \lambda_u (a_6^c - a_6^u) + \lambda_t a_6^c, \tag{5}$$

where the chiral factor $r = 2M_K^2/[(m_b + m_u)(m_s + m_u)]$. In the numerical calculations we set $m_u = \hat{m}$ and the coefficients a_1, a_4^u, a_4^c, a_6^u and a_6^c, evaluated at the scale $\mu = 2.1$ GeV, are taken from Table III of [21]. These coefficients take into account some QCD factorization corrections. We do not include small corrections coming from hard gluon exchanges with spectator-quark, from annihilation terms and from electroweak penguin diagrams. The charming penguin contribution can be parametrized as

$$C(m) = -(M_B^2 - m^2) f_\pi F_\pi (\lambda_u P_1^{\text{GIM}} + \lambda_t P_1), \tag{6}$$

where m is $m_{\pi\pi}$ or M_K, F_π is the $B \to \pi$ transition form factor calculated at the zero m_π^2 limit and P_1^{GIM}, P_1 are complex parameters. Determination of these parameters has been done in [21–24] by fitting some charmless two-body B-decay data.

The neutral $\bar{B}^0 \to (\pi^+\pi^-)_S K_S^0$ amplitude is similar to the charged one:

$$\langle (\pi^+\pi^-)_S K_S^0 | H | \bar{B}^0 \rangle = \frac{\langle (\pi^+\pi^-)_S K^- | H | B^- \rangle}{\sqrt{2}} \tag{7}$$

with $a_1 = 0$ and $m_u \to m_d$.

The amplitudes for B-decays into three kaons read

$$\begin{aligned}
&\langle (K^+K^-)_S K^- | H | B^- \rangle \\
&= \frac{G_F}{\sqrt{2}} \frac{1}{\sqrt{2}} \{ \chi [P(m_{K\bar{K}}) U + C(m_{K\bar{K}})] \\
&\quad \times [\Gamma_2^{n*}(m_{K\bar{K}}) + \Gamma_2^{n*}(\tilde{m}_{K\bar{K}})] \\
&\quad + [Q(m_{K\bar{K}}) V + \chi C(m_K)] \\
&\quad \times [\Gamma_2^{s*}(m_{K\bar{K}}) + \Gamma_2^{s*}(\tilde{m}_{K\bar{K}})] \}
\end{aligned} \tag{8}$$

and

$$\langle (K^+K^-)_S K_S^0 | H | \bar{B}^0 \rangle = \frac{\langle (K^+K^-)_S K^- | H | B^- \rangle}{\sqrt{2}} \tag{9}$$

with $a_1 = 0$, $\Gamma_2^{n,s*}(\tilde{m}_{K\bar{K}}) = 0$ and $m_u \to m_d$.

Here $m_{K\bar{K}}$ is the K^+K^- effective mass and $\tilde{m}_{K\bar{K}}$ is the effective mass with the second possible K^+K^- combination. $\Gamma_2^n(m_{K\bar{K}})$ and $\Gamma_2^s(m_{K\bar{K}})$ are the non-strange and strange kaon scalar form factors. Decay amplitudes with $(\pi^0\pi^0)_S$ and $(K^0\bar{K}^0)_S$ final states are the same as those with $(\pi^+\pi^-)_S$ and $(K^+K^-)_S$ pairs, respectively.

If one uses the a_i coefficients [19] $a_4^u = a_4^c = a_4$ and $a_6^u = a_6^c = a_6$, then our formula (1) with $C(m) = 0$ has the same algebraic structure as the $B^- \to \sigma\pi^-$ amplitude of Ref. [15] (see their Eq. (25)). The above equalities for a_i are valid in naive factorization but not in QCD factorization. A particular feature of the two penguin contributions to the $b \to s$ transition is the near cancellation of these two terms in Eq. (4) due to $a_4^c \approx a_6^c$ and $r \approx 1$. This has been pointed out by Chernyak in his estimation of the scalar production in B decays [25] and also by Gardner and Meißner [15].

Replacing the λ_u and λ_t values by their complex conjugate values λ_u^* and λ_t^* in Eqs. (1), (7)–(9) gives the amplitudes for the $B^+ \to (\pi^+\pi^-)_S K^+$, $B^0 \to (\pi^+\pi^-)_S K_S^0$, $B^+ \to (K^+K^-)_S K^+$ and $B^0 \to (K^+K^-)_S K_S^0$ decays.

3. Final state interactions

In the B decays considered above, one should include in the final states the $\pi\pi \to \pi\pi$ or the $K\bar{K} \to K\bar{K}$ rescattering and the $\pi\pi \to K\bar{K}$ or the $K\bar{K} \to \pi\pi$ transitions. The $(\pi\pi)_S$ and $(K\bar{K})_S$ pairs are formed from the $u\bar{u}, d\bar{d}$ and $s\bar{s}$ pairs. The four scalar form factors appearing in Eqs. (1) and (8) are defined [16] as

$$\begin{aligned}
\begin{pmatrix} \Gamma_1^n(m) \\ \Gamma_2^n(m) \end{pmatrix} &= \frac{1}{\sqrt{2}B_0} \begin{pmatrix} \langle 0|n\bar{n}|\pi\pi\rangle \\ \langle 0|n\bar{n}|K\bar{K}\rangle \end{pmatrix}, \\
\begin{pmatrix} \Gamma_1^s(m) \\ \Gamma_2^s(m) \end{pmatrix} &= \frac{1}{\sqrt{2}B_0} \begin{pmatrix} \langle 0|s\bar{s}|\pi\pi\rangle \\ \langle 0|s\bar{s}|K\bar{K}\rangle \end{pmatrix},
\end{aligned} \tag{10}$$

where m is the effective $\pi\pi$ or $K\bar{K}$ mass, $n\bar{n} = (\bar{u}u + \bar{d}d)/\sqrt{2}$ and $|0\rangle$ denotes the vacuum state. The final state interactions, which satisfy the unitarity constraints, are incorporated in the following formulae:

$$\Gamma_i^{n,s}(m) = R_i^{n,s}(m) + \sum_{j=1}^{2} \langle k_i | R_j^{n,s}(m) G_j(m) T_{ij}(m) | k_j \rangle, \tag{11}$$

where $|k_i\rangle$ and $|k_j\rangle$ represent the wave functions of two mesons in the momentum space and the indices $i, j = 1, 2$ refer to the $\pi\pi$ and $K\bar{K}$ channels, respectively. The center of mass channel momenta are given by $k_1 = \sqrt{m^2/4 - m_\pi^2}$ and $k_2 = \sqrt{m^2/4 - m_K^2}$. The matrix T is the two-body scattering matrix. Here we use the solution A of the $\pi\pi$ and $K\bar{K}$ coupled channel model [17]. As we restrict ourselves to $m_{\pi\pi} \lesssim 1.2$ GeV, the third effective $(2\pi)(2\pi)$ coupled channel considered in this model, with a threshold around 1.4 GeV, has a small effect. The functions $G_i(m)$ are the free Green's functions defined in [17] and $R_i^{n,s}(m)$ are the production functions responsible for the initial formation of the meson pairs prior to rescattering. The production functions have been derived by Meißner and Oller in the one-loop approximation of the chiral perturbation theory [16]. Using their Eqs. (37), (38), (41) and (44) one obtains

$$\begin{aligned}
R_1^n(m) &= 0.566 + 0.414 m^2, \\
R_2^n(m) &= -0.322 + 0.527 m^2, \\
R_1^s(m) &= -0.036 + 0.353 m^2, \\
R_2^s(m) &= 0.071 + 0.338 m^2,
\end{aligned} \tag{12}$$

where m is in GeV.

If one considers only the on-shell contributions in Eq. (11) then the scalar form factors can be written in terms of the phase shifts $\delta_{\pi\pi}(m)$, $\delta_{KK}(m)$ and of the inelasticities $\eta(m)$:

$$\Gamma_1^{n,s*}(m) = \frac{1}{2}\Big[R_1^{n,s}(m)\big(1+\eta(m)e^{2i\delta_{\pi\pi}(m)}\big) - iR_2^{n,s}(m) \times \sqrt{\frac{k_2}{k_1}}\sqrt{1-\eta^2(m)}e^{i[\delta_{\pi\pi}(m)+\delta_{K\bar{K}}(m)]}\Big], \tag{13}$$

$$\Gamma_2^{n,s*}(m) = \frac{1}{2}\Big[R_2^{n,s}(m)\big(1+\eta(m)e^{2i\delta_{K\bar{K}}(m)}\big) - iR_1^{n,s}(m) \times \sqrt{\frac{k_1}{k_2}}\sqrt{1-\eta^2(m)}e^{i[\delta_{\pi\pi}(m)+\delta_{K\bar{K}}(m)]}\Big]. \tag{14}$$

Below the $K\bar{K}$ threshold $\eta(m)=1$ and

$$\Gamma_1^{n,s*}(m) = R_1^{n,s}(m)\cos\delta_{\pi\pi}(m)e^{i\delta_{\pi\pi}(m)}. \tag{15}$$

Particularly interesting is the $m_{\pi\pi}$ range where the phase shifts $\delta_{\pi\pi}$ are close to 180°. Then one expects a *maximum* in $|\Gamma_1^{n,s}|$. As we shall see in the next section, this is the case for the production of the $f_0(980)$ resonance. Note that the $\Gamma_1^{n,s}$ is zero when $\delta_{\pi\pi}=\pi/2$.

The off-shell contributions to the form factors are very much model dependent and are not considered in the following calculations.

4. Results

The amplitudes for the $B\to(\pi\pi)_S K$ decays considered in Section 2 depend only on the effective mass $m_{\pi\pi}$. Integrating on the Dalitz plot over the kinematically allowed range of $m_{\pi K}$, one obtains the differential $B\to\pi\pi K$ decay distribution

$$\frac{d\Gamma}{dm_{\pi\pi}} = \frac{m_{\pi\pi}k_1 p_K}{4M_B^3(2\pi)^3}\big|\mathcal{M}\big(B\to(\pi\pi)_S K\big)\big|^2, \tag{16}$$

where k_1 and $p_K=\sqrt{E_K^2(m_{\pi\pi})-m_K^2}$ are the pion and kaon momenta in the $\pi\pi$ center of mass system, $E_K(m_{\pi\pi})=\frac{1}{2}(M_B^2-m_{\pi\pi}^2-m_K^2)/m_{\pi\pi}$ being the corresponding kaon energy. In (16) $\mathcal{M}$ denotes the decay amplitude given by Eq. (1) or (7). Dividing $d\Gamma/dm_{\pi\pi}$ by the appropriate B^+ or B^0 total width Γ_B one obtains the differential branching ratio $d\mathcal{B}/dm_{\pi\pi}$.

Before presenting our results we fix the constants which appear in the formulae for the decay amplitudes (1), (7)–(9). The masses of pions, kaons, B-mesons and their life times, the values of the Fermi coupling constant G_F and the kaon decay constant $f_K=0.1598$ GeV are taken from [1]. For the kaon mass we use the average of the charged and neutral ones. Following the results obtained in [26] and applied in [15] we use $F_0^{B\to(\pi\pi)_S}(M_K^2)=0.46$ for the $B\to(\pi\pi)_S$ transition form factor.

The $B\to K$ transition form factor is approximated by a constant equal to 0.39 which is close to the number $F_0^{B\to K}(0)=0.379$ quoted by Bauer, Stech and Wirbel in Table 14 of [18]. This approximation is justified since we consider a relatively narrow range of the effective masses $m_{\pi\pi}$. These masses are much smaller than the mass of the heavy excited B meson used in polar models of the transition form factor. So we fix $F_0^{B\to K}(m_{\pi\pi}^2)\approx F_0^{B\to K}(m_{f_0}^2)$ where m_{f_0} is the $f_0(980)$ mass. The value 0.379 quoted in [18] as well as a more recent value of 0.33, obtained by Ball and Zwicky in [27], are within the limits given by Beneke and Neubert in Table 1 of [28] for $F_0^{B\to K}(0)=0.34\pm0.05$.

Using the Wolfenstein representation [29], the CKM matrix elements are written in a form accurate to the level of λ^6 [30]. The values of the parameters, taken from the CKMfitter Group, are: $\lambda\equiv V_{us}=0.2265$, $\bar{\rho}=0.189$, $\bar{\eta}=0.358$ and $A=0.801$ [31].

The constant χ will be fitted to the experimental branching ratio of the decay $B^+\to f_0(980)K^+$. Once fixed it will be used to make absolute model predictions in the whole $m_{\pi\pi}$ range studied here for the $B^\pm\to\pi^+\pi^-K^\pm$ reaction and for other decays like $B^0\to\pi^+\pi^-K_S^0$, $B^+\to K^+K^-K^+$ and $B^0\to K^+K^-K_S^0$. The value of χ is, however, not arbitrary. It can be estimated from the following considerations. First we shall concentrate ourselves on the $m_{\pi\pi}$ range close to the relatively narrow resonance $f_0(980)$ clearly visible in the B decays into $\pi^+\pi^-K$. The $f_0(980)$ decays mainly into $\pi\pi$. The coupling constant of $f_0(980)$ to the $\pi\pi$ pair can be approximated by

$$g_{f_0\pi\pi} = m_{f_0}\sqrt{\frac{8\pi\Gamma(f_0\to\pi\pi)}{k_1(m_{f_0})}} \tag{17}$$

(see, for example, Eq. (36) of [32]). Here $\Gamma(f_0\to\pi\pi)$ is the $\pi\pi$ partial width of the $f_0(980)$ and $k_1(m_{f_0})$ is the pion momentum in the $f_0(980)$ rest frame. The two scalar form factors $\Gamma_1^n(m_{\pi\pi})$ and $\Gamma_1^s(m_{\pi\pi})$ are strongly peaked at the $f_0(980)$ mass;

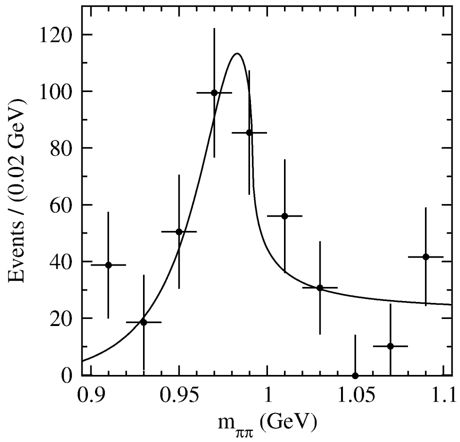

Fig. 2. Effective $\pi^+\pi^-$ mass distribution in $B^\pm \to \pi^+\pi^-K^\pm$ decays. The BaBar data are taken from [5]. The solid line results from our model.

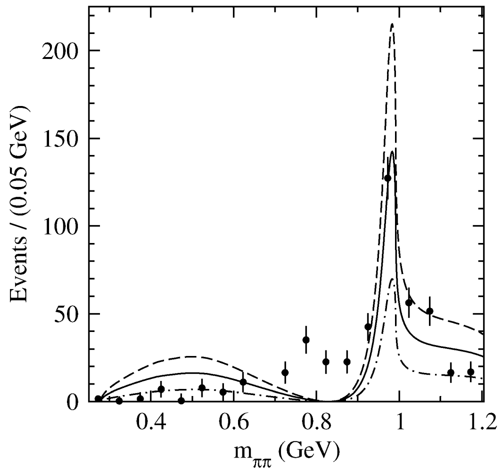

Fig. 3. Comparison of the Belle [7] $\pi^+\pi^-$ effective mass distribution for the $B^\pm \to \pi^+\pi^-K^\pm$ decays with our model. The dashed line corresponds to the $B^+ \to \pi^+\pi^-K^+$ decays, the dotted-dashed line is for the $B^- \to \pi^+\pi^-K^-$ decays and the solid line is the average for the B^+ and B^- decays.

one can show, however, using Eqs. (12) and (13) that $|\Gamma_1^n(m_{f_0})| \gg |\Gamma_1^s(m_{f_0})|$. Therefore one can write the approximate relation

$$\chi = \frac{g_{f_0\pi\pi}}{m_{f_0}\Gamma_{\text{tot}}(f_0)} \frac{1}{|\Gamma_1^n(m_{f_0})|} \tag{18}$$

(see, for instance, Eq. (35) of [15] for the $f_0(600)$ case). Here $\Gamma_{\text{tot}}(f_0)$ is the total $f_0(980)$ width. In Ref. [32] $\Gamma_{\text{tot}}(f_0) = (71 \pm 14)$ MeV. For $m_{f_0} = 0.98$ GeV we obtain from Eq. (13) $|\Gamma_1^n(m_{f_0})| \approx 0.96$. If $\Gamma_{\text{tot}}(f_0) \approx \Gamma(f_0 \to \pi\pi) = 60$ MeV then Eqs. (17) and (18) give $\chi \approx 30\ \text{GeV}^{-1}$.

In this Letter we do not attempt to make our own fits to data by adjusting the constants P_1^{GIM} and P_1 in the function $C(m)$ of (6). We present the results obtained with the charming penguin amplitudes determined in [21,24]. The theoretical curves shown in Figs. 2–5 correspond to the first set of amplitudes.

4.1. $B^\pm \to \pi^+\pi^-K^\pm$ decays

In Figs. 2 and 3 we show a comparison of the $\pi\pi$ effective mass distributions for the $B^\pm \to \pi^+\pi^-K^\pm$ decays calculated in our model with the results obtained by BaBar [5] and Belle [7].

The branching fraction $\mathcal{B}(B^\pm \to f_0(980)K^\pm, f_0(980) \to \pi^+\pi^-) = (9.2 \pm 1.2^{+2.1}_{-2.6}) \times 10^{-6}$ obtained by BaBar [5] can be reproduced in our model for the value of $\chi = 35.0\ \text{GeV}^{-1}$. This value is close to our estimation given above. The Belle Collaboration has reported a slightly smaller value of the branching ratio $(7.55 \pm 1.24 \pm 0.69^{+1.48}_{-0.96}) \times 10^{-6}$ [7]. Both values are compatible within their error bars. The average value given by the Heavy Flavor Averaging Group (HFAG) is equal to $(8.49^{+1.35}_{-1.26}) \times 10^{-6}$ [33]. For this value the constant χ is 33.5 GeV^{-1} when we use the charming penguin amplitudes of [21].

In Fig. 2 the normalization of the theoretical curve to the data in the $f_0(980)$ range is based on the total number of events seen by BaBar for $m_{\pi\pi}$ from 0.9 to 1.1 GeV, multiplied by a correction factor of 0.92 being a ratio of the branching ratios 8.49×10^{-6} (HFAG's value) and 9.2×10^{-6} (BaBar's value).

In Fig. 3 the solid curve is normalized at $m_{\pi\pi} = 976$ MeV. This corresponds to the maximum of the background subtracted mass distribution. We calculate it from Fig. 9(e) of [7] as 133 events/50 MeV. This number is obtained by taking into account two factors. The first factor, equal to 92%, follows from the fraction of the $f_0(980)$ components in the full spectrum resulting from the solution 1 of the Belle model, called $K\pi\pi - C_0$ [7]. The second factor, equal to 1.125, comes from the ratio of the above mentioned branching ratios: 8.49×10^{-6} and 7.55×10^{-6}.

One can see from Fig. 2 that our model describes quite well the $\pi^+\pi^-$ spectrum measured by BaBar in vicinity of $f_0(980)$. The model depicts also a very pronounced maximum seen by Belle near 1 GeV (Fig. 3). This maximum is attributed to the $f_0(980)$ resonance. At lower $\pi\pi$ masses near 500 MeV one can notice a broad theoretical maximum which we can relate to the σ or $f_0(600)$ meson. Experimental data in this mass range are not in disagreement with this feature of our model although the present errors are too large to draw a definite conclusion supporting the evidence of $f_0(600)$ in the $B^\pm \to \pi^+\pi^-K^\pm$ decay. Also the preliminary data of the BaBar Collaboration [6] below 600 MeV show some enhancement of the $\pi^+\pi^-K^\pm$ events over the background. Let us remark that both collaborations have not included the σ meson in their fits to data. Integrating the $\pi\pi$ spectrum between the $\pi^+\pi^-$ threshold and 700 MeV we find the average branching ratio for the $B^+ \to \sigma K^+$ and $B^- \to \sigma K^-$ decays equal to 3.9×10^{-6}. A surplus of events near 0.8 GeV is due to the $B^\pm \to \rho^0 K^\pm$ decay which are not taken into account in this model since we concentrate ourselves on the S-wave $\pi^+\pi^-$ events.

In Figs. 2 and 3 we have shown the averaged $\pi^+\pi^-$ spectra of the two decays: $B^+ \to \pi^+\pi^-K^+$ and $B^- \to \pi^+\pi^-K^-$. They have been calculated for the charming penguin amplitudes $C(m)$ of Eq. (6) fitted in Ref. [21] by $P_1 = (0.068 \pm 0.007)\exp[i(1.32 \pm 0.10)]$ and $P_1^{\mathrm{GIM}} = (0.32 \pm 0.14)\exp[i(1.0 \pm 0.27)]$. Note that the strength of the P_1 contribution to $C(m)$ is about ten times larger than that of P_1^{GIM}. If we neglect both of them the branching ratios are smaller by a factor of about 4.

Using these long distance contributions we have found a very pronounced direct CP asymmetry in the $\pi^+\pi^-$ spectra. There are many more decays of B^+ into $\pi^+\pi^-K^+$ than of B^- into $\pi^+\pi^-K^-$ (see Fig. 3) This asymmetry for the above choice of parameters is even higher than the direct CP asymmetry recently found in the $B^0 \to K^+\pi^-$ and $\bar{B}^0 \to K^-\pi^+$ decays [34,35]. The charge asymmetry is defined as

$$\mathcal{A}_{CP} = \frac{\frac{d\Gamma(B^-\to\pi^+\pi^-K^-)}{dm_{\pi\pi}} - \frac{d\Gamma(B^+\to\pi^+\pi^-K^+)}{dm_{\pi\pi}}}{\frac{d\Gamma(B^-\to\pi^+\pi^-K^-)}{dm_{\pi\pi}} + \frac{d\Gamma(B^+\to\pi^+\pi^-K^+)}{dm_{\pi\pi}}}. \tag{19}$$

If we integrate $d\Gamma/dm_{\pi\pi}$ between 0.9 and 1.1 GeV then $\mathcal{A}_{CP} = -0.52 \pm 0.12$. The errors come from the uncertainties of the charming penguin amplitudes determined in [21]. It would be very useful to confront this number with a future experimental determination of this asymmetry in the $B^\pm \to f_0(980)K^\pm$ decays.

The charge CP violating asymmetry is very sensitive to the magnitude and the phase of the charming penguin contribution. Using the different approach of [22] and the fits presented in [24] for the $B \to K\pi$ decays the value $P_1 = (0.08 \pm 0.02)\exp[-i(0.6 \pm 0.5)]$ has been obtained, while P_1^{GIM} has not been determined. With this value of P_1 and with $P_1^{\mathrm{GIM}} = 0$, one obtains a good agreement with the HFAG branching fraction for $\chi = 23.5\ \mathrm{GeV}^{-1}$. Then the charge asymmetry is positive and equal to 0.20 ± 0.20. We see that the different charming penguin amplitudes fitted to data are not in mutual agreement. However, the two analyses are based on different data sets. In [21] 18 different branching ratios have been fitted and in [24] the fit has been performed on 8 observables for the $B \to K\pi$ decays.

In the following subsections the predictions for the two penguin amplitudes are given without any readjustment of the constants χ.

4.2. $B^0 \to \pi^+\pi^-K^0$ decays

In Fig. 4 we show the comparison of the model predictions for the neutral B decays with the BaBar results [4]. Here the experimental background contribution is added to the theoretical part calculated from Eq. (7). The average branching ratio for the B^0 and $\bar{B}^0$ decays into $(\pi^+\pi^-)_S K_S^0$, for the $\pi^+\pi^-$ mass between 0.85 and 1.1 GeV, equals to 2.93×10^{-6}.

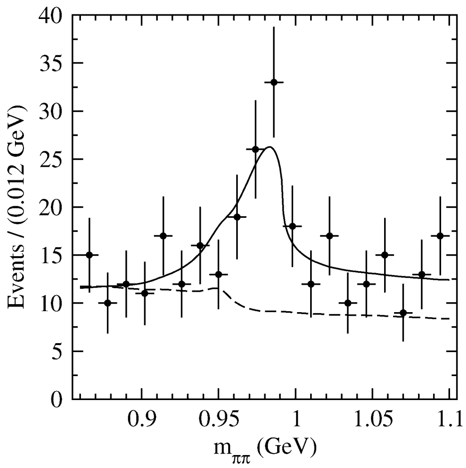

Fig. 4. Effective $\pi^+\pi^-$ mass distribution for the $B^0 \to \pi^+\pi^-K^0$ decays. The BaBar data and the dashed line corresponding to the background, are taken from [4]. The solid line is our model result.

Twice this value compares well with the experimental determination of the BaBar Collaboration $\mathcal{B}(B^0 \to f_0(980)K^0) \times \mathcal{B}(f_0(980) \to \pi^+\pi^-) = (6.0 \pm 0.9 \pm 0.6 \pm 1.2) \times 10^{-6}$. Our curve in Fig. 4 is normalized to the total number of events multiplied by the ratio of $2 \times 2.93/6.0 = 0.98$.

The branching ratio diminishes by a factor of 18 if the charming penguins amplitudes are omitted. This was expected due to the near cancellation, mentioned in Section 2, between the two penguin diagram contributions. The absence of tree diagram in neutral B decays explains the difference with the charged B decays where the branching ratio drops by a factor of 4 if the charming penguin terms are not present.

The direct CP violation asymmetry between the decay of B^0 and $\bar{B}^0$ into $(\pi^+\pi^-)_S K_S^0$ defined as

$$\mathcal{A} = \frac{\mathcal{B}(\bar{B}^0 \to \pi^+\pi^-\bar{K}^0) - \mathcal{B}(B^0 \to \pi^+\pi^-K^0)}{\mathcal{B}(\bar{B}^0 \to \pi^+\pi^-\bar{K}^0) + \mathcal{B}(B^0 \to \pi^+\pi^-K^0)} \tag{20}$$

is much smaller than the asymmetry (19) for the $B^\pm \to (\pi^+\pi^-)_S K^\pm$ decays. It amounts to 0.01 ± 0.10 when calculated with the charming penguin parameters of [21] in the $m_{\pi\pi}$ range between 0.85 and 1.1 GeV. The reason of its smallness is due to the absence of the tree diagram contribution (Fig. 1(a)) for the B^0 or $\bar{B}^0$ decays.

The BaBar [3] and Belle [8] values for this asymmetry, $\mathcal{A} = 0.24 \pm 0.31 \pm 0.15$ and $\mathcal{A} = -0.39 \pm 0.27 \pm 0.08$, respectively, have large experimental errors and agree with our result. The HFAG average is -0.14 ± 0.22 [33]. In Fig. 5 we compare the $\pi^+\pi^-$ spectrum presented by Chen on behalf of the Belle Collaboration for the $B^0 \to \pi^+\pi^-K_S^0$ decays [9] with our calculation when the background is subtracted. Our curve is normalized to the number of 94 events attributed to the $f_0(980)K_S^0$ decay in the $m_{\pi\pi}$ range between 0.89 and 1.088 GeV [8]. The Belle experimental determination of the branching ratio for the $B^0 \to \pi^+\pi^-K_S^0$ decay is not yet available. As in the $B^\pm \to \pi^+\pi^-K^\pm$ case, shown in Fig. 3, we predict some σ contribution of the decay amplitude in the low $\pi^+\pi^-$ range below the position of $\rho(770)$ enhancement visible in the data. The zero value of our spectrum near $m_{\pi\pi} = 0.8$ GeV, also present in Fig. 3, comes from $\delta_{\pi\pi} = \pi/2$ (see Eq. (15)).

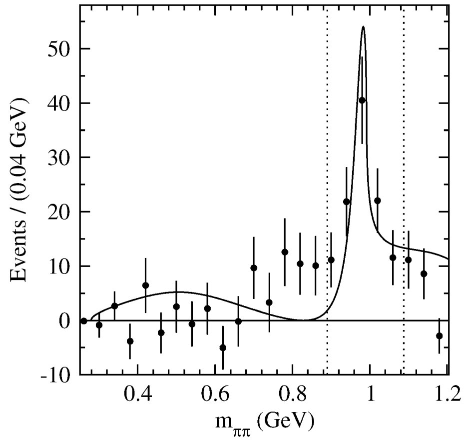

Fig. 5. Comparison of the Belle data [9] (after a background subtraction) with our model (solid line) for the $B^0 \to \pi^+\pi^-K_S^0$ decays. Dotted vertical lines delimit a band of the $f_0(980)$ events used in the curve normalization.

We have also calculated the time-dependent CP asymmetry in the neutral B^0 decays into $\pi^+\pi^-K_S^0$:

$$A(t) = \frac{\frac{d\Gamma(\bar{B}^0\to\pi^+\pi^-K_S^0)}{dt} - \frac{d\Gamma(B^0\to\pi^+\pi^-K_S^0)}{dt}}{\frac{d\Gamma(\bar{B}^0\to\pi^+\pi^-K_S^0)}{dt} + \frac{d\Gamma(B^0\to\pi^+\pi^-K_S^0)}{dt}}. \tag{21}$$

The time dependence of this asymmetry can be approximated as

$$A(t) = \mathcal{S}\sin(\Delta m t) + \mathcal{A}\cos(\Delta m t), \tag{22}$$

where Δm is the difference between the masses of the heavy and light B meson eigenstates.

In the $f_0(980)$ mass range between 0.85 and 1.1 GeV the asymmetry parameter $\mathcal{S}$ equals to -0.63 ± 0.09. This result corresponds to the charming penguin parameters of [21]. The BaBar result is $\mathcal{S} = -0.95^{+0.32}_{-0.23} \pm 0.10$ [3] and the Belle number $\mathcal{S} = +0.47 \pm 0.41 \pm 0.08$ [8]. The two experimental results are not in agreement with each other but their experimental errors are large. Our result agrees better with the BaBar value. The HFAG [33] gives the average $\mathcal{S} = -0.39 \pm 0.26$ which is in agreement with our prediction of -0.63. The value of $\mathcal{S}$ for the charming penguin amplitudes of [24], considered in Section 4.1, is -0.77.

4.3. $B \to K\bar{K}K$ decays

We have calculated the $(K^+K^-)_S$ spectra and the branching ratios for the $B^+ \to (K^+K^-)_S K^+$

and $B^- \to (K^+K^-)_S K^-$ decays. For the charming penguin amplitudes of [21] one obtains a large direct CP violating asymmetry of -0.44 ± 0.12 in the K^+K^- mass range between the threshold and 1.1 GeV. The average branching ratio for the above mass range equals to $(1.8 \pm 0.4) \times 10^{-6}$. This value is below the upper limit of 2.9×10^{-6} found by Garmash et al. for the branching fraction $\mathcal{B}(B^+ \to f_0(980)K^+, f_0(980) \to K^+K^-)$ [7]. The theoretical $(K^+K^-)_S$ spectrum is flat in the range between 1.0 and 1.2 GeV and agrees well with the experimental distribution shown in Fig. 13(d) of [7]. As in the case of the $B^\pm \to \pi^+\pi^-K^\pm$ decays the asymmetry for the $B^\pm \to (K^+K^-)_S K^\pm$ process strongly depends on the charming penguin amplitude. With the amplitudes of [24] considered in Section 4.1, the asymmetry is positive and equal to 0.29 ± 0.21, the average branching ratio being $(1.7 \pm 0.7) \times 10^{-6}$. In these calculations we neglect the symmetrized form factors $\Gamma_2^{n,s}(\tilde{m}_{K\bar{K}})$ in Eq. (8) assuming that for small masses of $m_{K\bar{K}}$ the $\tilde{m}_{K\bar{K}}$ masses are sufficiently high and that the Γ_2 form factors decrease rapidly with increasing $\tilde{m}_{K\bar{K}}$.

For the $B^0 \to (K^+K^-)_S K_S^0$ and $\bar{B}^0 \to (K^+K^-)_S K_S^0$ decays we have obtained very small direct CP violating asymmetries $\mathcal{A} = 0.01 \pm 0.10$ and $\mathcal{A} = 0.001 \pm 0.001$ for the penguin amplitudes of [21] and [24], respectively. These numbers agree well with the experimental findings of Belle [8] $(-0.08 \pm 0.12 \pm 0.07)$ and BaBar [36] $(-0.10 \pm 0.14 \pm 0.04)$. The parameter $\mathcal{S}$, equal to -0.64 or -0.77, depending on the set of penguin amplitudes, is also in general agreement with the results of $-0.74 \pm 0.27^{+0.19}_{-0.39}$ and $-0.55 \pm 0.22 \pm 0.04 \pm 0.11$ reported by Belle [8] and BaBar [36], respectively.

Within our model one finds the same asymmetries for the $B^0 \to K_S^0 K_S^0 K_S^0$ decay as for the $B^0 \to (K^+K^-)_S K_S^0$ process provided that the S-wave is dominant in the production of the $K_S^0 K_S^0$ pairs and their effective masses are not large. The new data of Belle [37] ($\mathcal{A} = 0.54 \pm 0.34 \pm 0.09$, $\mathcal{S} = 1.26 \pm 0.68 \pm 0.20$) and BaBar [38] ($\mathcal{A} = 0.34^{+0.25}_{-0.28} \pm 0.05$, $\mathcal{S} = -0.71^{+0.38}_{-0.32} \pm 0.04$) agree for $\mathcal{A}$ and disagree for $\mathcal{S}$. Our results for $\mathcal{S}$ are close to the BaBar value, however the experimental errors of both collaborations are still too large to make a definite conclusion.

5. Summary and outlook

We have analyzed the charged and neutral three-body B meson decays into the $\pi^+\pi^-K$, K^+K^-K and $K_S^0 K_S^0 K_S^0$ systems. Our primary aim is a construction of decay amplitudes including the $\pi^+\pi^-$ final state interactions in a rather wide range of effective masses between the $\pi\pi$ threshold and 1.2 GeV. The model is based on a factorization approximation with some QCD corrections and contains the dominant charming penguin terms. Using a single amplitude, we are able to describe simultaneously the production of two scalar-isoscalar resonances $f_0(600)$ and $f_0(980)$. The B decay amplitudes to the $\pi\pi K$ and $K\bar{K}K$ states are connected as the coupling between the $\pi\pi$ and $K\bar{K}$ channels above 1 GeV is incorporated in our model. No adjustable free parameters, like arbitrary phase factors between contributions of different resonances, are needed. We have obtained a good agreement with most of the recent BaBar and Belle data for the $\pi\pi$ effective mass distributions, the branching ratios and the time-dependent CP violating asymmetries. Our results are summarized in Table 1. These numbers depend only weakly on the choice of the renormalization scale, the corresponding quark masses m_b and m_s, and the value of $F_0^{B\to K}(0)$. The changes are smaller than the errors in the determination of the charming penguin amplitudes.

If we use the charming penguin amplitudes determined in [21] then the direct CP violation asymmetry in the charged B decays to $\pi\pi K$ is strongly negative (~ -0.5). However, for the charming penguin amplitudes taken from [24] this asymmetry is positive ($\sim +0.2$). A similar difference is found for the $B^\pm \to K^+K^-K^\pm$ reactions. Future independent measurements of the $B^\pm \to \pi^+\pi^-K^\pm$ and $B^\pm \to K^+K^-K^\pm$ asymmetries will be crucial for a decisive test of the phase of the long distance contributions. Let us stress the importance of charming penguin amplitudes. If we omit them then the average branching ratio for the $B^0 \to \pi^+\pi^-K_S^0$ decay is too small by a factor of 18 and that of the $B^\pm \to \pi^+\pi^-K^\pm$ mode by a factor of 4. Even an *ad hoc* adjustment of the constant χ to fit the experimental charged B decays does not allow one to explain the neutral B decays. However, when the charming penguin amplitudes are included, we get a good agreement for both channels for

Table 1
Average branching fractions $\mathcal{B}$ in units of 10^{-6}, asymmetries $\mathcal{A}_{CP}$, $\mathcal{A}$ and $\mathcal{S}$ of our model compared to the average values of HFAG [33]. The $m_{\pi\pi}$ mass ranges for the $B^{\pm} \to f_0(980)K^{\pm}$ and for the $B^0 \to f_0(980)K^0$ decays are $(0.9, 1.1)$ GeV and $(0.85, 1.1)$ GeV, respectively. The upper limit of the $(K^+K^-)_S$ or $(K_S^0K_S^0)_S$ effective mass is 1.1 GeV. The model errors come from the uncertainties of the charming penguin amplitudes $C(m)$ (Eq. (6)) determined in the fits of [21] (model I) or [24] (model II). The experimental errors for $\mathcal{S}$ in the $B^0 \to (K^+K^-)_S K_S^0$ decay are the statistical ones

B decay mode		Average HFAG's values	Model I $\chi = 33.5$ GeV^{-1}	Model II $\chi = 23.5$ GeV^{-1}
$B^{\pm} \to f_0(980)K^{\pm}$, $f_0 \to \pi^+\pi^-$	$\mathcal{B}$	$8.49^{+1.35}_{-1.26}$	8.49 (fit)	8.46 (fit)
	$\mathcal{A}_{CP}$	no data	-0.52 ± 0.12	0.20 ± 0.20
$B^0 \to f_0(980)K^0$, $f_0 \to \pi^+\pi^-$	$\mathcal{B}$	6.0 ± 1.6	5.9 ± 1.6	5.8 ± 2.8
	$\mathcal{A}$	-0.14 ± 0.22	0.01 ± 0.10	0.0004 ± 0.0010
	$\mathcal{S}$	-0.39 ± 0.26	-0.63 ± 0.09	-0.77 ± 0.0004
$B^{\pm} \to (K^+K^-)_S K^{\pm}$	$\mathcal{B}$	< 2.9 [7]	1.8 ± 0.4	1.7 ± 0.7
	$\mathcal{A}_{CP}$	no data	-0.44 ± 0.12	0.29 ± 0.21
$B^0 \to (K^+K^-)_S K_S^0$	$\mathcal{B}$	no data	1.1 ± 0.3	1.2 ± 0.5
	$\mathcal{A}$	-0.09 ± 0.10	0.01 ± 0.10	0.001 ± 0.001
	$\mathcal{S}$	-0.55 ± 0.22 [36] -0.74 ± 0.27 [8]	-0.64 ± 0.09	-0.77 ± 0.0006
$B^0 \to (K_S^0K_S^0)_S K_S^0$	$\mathcal{B}$	no data	1.1 ± 0.3	1.2 ± 0.5
	$\mathcal{A}$	0.41 ± 0.21	0.01 ± 0.10	0.001 ± 0.001
	$\mathcal{S}$	-0.26 ± 0.34	-0.64 ± 0.09	-0.77 ± 0.0006

χ values close to the estimation based on the $f_0(980)$ properties.

The model presented in this Letter can be extended to larger effective $\pi\pi$ mass range, in particular to the range where the $f_0(1500)$ is important. One can also include the final state interactions between one of the pions and the kaon in the B or D decays to the $\pi\pi K$ system. Especially interesting are the π^-K^+ or π^+K^- subsystems where scalar and vector resonances can play an important role.

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